# MAR IVANIOS COLLEGE (AUTONOMOUS)

**Affiliated to the University of Kerala,**
**Thiruvananthapuram**
**Kerala**

## SCHEME AND SYLLABUS
## FOR THE POSTGRADUATE PROGRAMME
## MASTER OF SCIENCE IN MATHEMATICS

**(With effect from 2021 Admissions)**

**Approved by the Board of Studies in**
**Mathematics and Statistics**

# PREAMBLE

As part of our continued efforts to familiarize the students with classical results as well as the latest advances, research outputs and applications in various branches of Mathematics, the Department of Mathematics of Mar Ivanios College (Autonomous) decided to revise the syllabus of MSc Mathematics with effect from the academic year 2021-22 onwards. Several levels of initial discussions and consultations were conducted for more than a year, both at the faculty level and also with external experts. The Board of Studies in Mathematics and Statistics held several informal consultations previously. After deliberations and incorporating the suggestions of external experts of the Board, the draft syllabus was approved at the meeting of the Board held on 4th March 2021. A major decision of the Board has been to add one more course each in Semester 1 and 2, increasing the number of courses in each of these semesters to 5 and enhancing the total maximum marks of the MSc programme to 2000. The Board has given emphasis to include more application level courses in the syllabus. Maximum effort, as far as possible, has been taken to avoid duplication with the existing UG Syllabus. The UGC-CSIR NET syllabus and syllabi of other reputed universities and colleges in India and abroad were also given due consideration. The final syllabus was circulated among the members and their approval was obtained. The Chairman and Members of the Board of Studies in Mathematics and Statistics would like to place on record their gratitude to the entire faculty who took part in the discussion and contributed to the design of the syllabus, which will be effective from the academic year 2021-22. The contributions of the eminent external experts of the Board, whose names are appended, are invaluable. We also specially remember the guidance given by Prof M. Thamban Nair, Professor, IIT Madras and Dr Viji Z Thomas, Associate Professor, IISER, Trivandrum in the design and revision of Linear and Abstract Algebra courses.

Comments and suggestions for further improvements and updations are welcome.

We submit the revised syllabus to the student community

Fr. Dr. Gigi Thomas
Chairman BoS
(Mathematics and Statistics)
Mar Ivanios College (Autonomous),
Thiruvananthapuram

Thiruvananthapuram
10-05-2021

## MAR IVANIOS COLLEGE (AUTONOMOUS), THIRUVANANTHAPURAM

## BOARD OF STUDIES IN MATHEMATICS & STATISTICS, 2020 – 2022

| No | Name | Designation |
|---|---|---|
| 1 | Fr. Dr. Gigi Thomas (Chairman) | Associate Professor & Head,<br>Department of Mathematics<br>Mar Ivanios College (Autonomous). |
| 2 | Prof. Dr. G. Suresh Singh<br>(University Nominee) | Professor & Head Department of Mathematics University of Kerala. |
| 3 | Prof. Dr. Raju George | Senior Professor Department of Mathematics<br>Dean (R&D), Dean (Students' Welfare)<br>Indian Institute of Space Science and Technology (IIST)Thiruvananthapuram. |
| 4 | Prof. Dr. M. P. Rajan | Professor<br>School of Mathematics<br>Indian Institute of Science Education and Research(IISER), Thiruvananthapuram. |
| 5 | Prof. Dr. Subrahamanian Moosath K. S. | Professor<br>Department of Mathematics<br>Indian Institute of Space Science and Technology (IIST)Thiruvananthapuram. |
| 6 | Prof. Dr. C. Satheesh Kumar | Professor & Head<br>Department of Statistics<br>University of Kerala. |
| 7 | Dr. Tony Thomas | Professor<br>Indian Institute of Information Technology andManagement Kerala (IIITMK)<br>Thiruvananthapuram. |
| 8 | Dr. S. H. S. Dharmaja | Associate Professor & Head<br>Department of Statistics Government College for WomenThiruvananthapuram. |
| 9 | Dr. Vishnu Namboothiri K. | Associate Professor<br>Department of Mathematics<br>Government College, Ambalappuzha. |
| 10 | Dr. K. R. Arun | Assistant Professor Grade I, School of Mathematics<br>Indian Institute of Science Education and Research (IISER), Thiruvananthapuram. |

| | | |
|---|---|---|
| 11 | Dr. Manil T. Mohan | Assistant Professor<br>Department of Mathematics<br>Indian Institute of Technology (IIT)<br>Roorkee. |
| 12 | Mr. Deepak Negi | Head<br>Applied Mathematics Division<br>Vikram Sarabhai Space Centre (VSSC)<br>Thiruvananthapuram. |
| 13 | Ms. Jyothi Ramaswamy | Head, Cyber Security wing<br>Tata Consultancy Services (TCS)<br>Trivandrum. |
| 14 | Dr. T. R. Sivakumar | Associate Professor (Rtd.) and Emeritus Faculty Department of Mathematics<br>Deputy Controller of Examinations<br>Mar Ivanios College (Autonomous). |
| 15 | Dr. K. L. Anandavally | Associate Professor and Head (Rtd.)<br>Department of Mathematics<br>Director - Self-Financing Programmes<br>Mar Ivanios College (Autonomous). |
| 16 | Mr. Sumesh S. S. | Assistant Professor<br>Department of Mathematics<br>Mar Ivanios College (Autonomous). |
| 17 | Ms. Tiji Thomas | Assistant Professor<br>Department of Mathematics<br>Mar Ivanios College (Autonomous). |
| 18 | Dr. Jill K. Mathew | Assistant Professor<br>Department of Mathematics<br>Mar Ivanios College (Autonomous). |
| 19 | Dr. Linda J. P. | Assistant Professor<br>Department of Mathematics<br>Mar Ivanios College (Autonomous). |
| 20 | Dr. Neeradha C. K. | Assistant Professor<br>Department of Mathematics<br>Mar Ivanios College (Autonomous). |

## EVALUATION AND ASSESSMENT

The Post Graduate programme M. Sc. Mathematics shall extend over a period of two academic years comprising of four semesters, each of 450 hours in 18 weeks duration. The syllabus and scheme of examinations of the programme are detailed below. The syllabus is effective from 2021 admission.

### 1. Evaluation

**a)** Evaluation of each course shall be done in two parts – Continuous Evaluation [CE]/ Continuous Assessment (CA) and End Semester Evaluation [ESE]/ End Semester Assessment (ESA).

**b)** The distribution of marks for the above two shall be as follows:

I. Continuous Evaluation CE /CA - 25% of the total marks for the course

II. End Semester Evaluation ESE/ESA - 75% of the total marks for the course

**c)** All documents/records of Continuous Evaluation shall be kept in the Department of Mathematics and shall be made available for verification by any competent authority, if and when necessary.

### 2. Continuous Evaluation

**a)** The allocation of marks for each component under Continuous Evaluation shall be as given below:

**Theory courses**

| | | |
|---|---|---|
| 1 | Attendance | 5 marks |
| 2 | Assignment | 5 marks |
| 3 | Seminar | 5 marks |
| 4 | Internal test(s) | 10 marks |
| | **Total** | **25 marks** |

**Practical**

| | | |
|---|---|---|
| 1 | Attendance | 5 marks |
| 2 | Internal test(s) | 10 marks |
| 3 | Practical Record | 10 marks |
| | **Total** | **25 marks** |

**b)** There shall be no Continuous Evaluation for the dissertation/project work.

### 3. Attendance

**a)** Students have to secure a minimum of 75% aggregate attendance within a semester to become eligible to register for each End Semester Examination. The attendance percentage will be calculated from the day of commencement of the semester to the last working day of that semester as specified in the Academic Calendar. Periodic evaluation of each student'sattendance shall be done by the respective Faculty Adviser within each semester.

**b)** In general, there will be no provision for condonation. However, in cases of extreme necessitystudents can approach the competent authority.

**c)** Reappearance of course(s) will be distinctly indicated in the final mark/grade sheet.

### d) Allotment of marks

The allotment of marks for attendance shall be as follows:

| Attendance (in %) | Marks (out of 5) |
|---|---|
| > 90 | 5 |
| >85 & ≤90 | 4 |
| >80 & ≤85 | 3 |
| >75 & ≤80 | 2 |
| 75 | 1 |
| < 75 | 0 |

## 4. Assignments:

**a)** Each student shall be required to do one assignment for each course in each semester.

**b)** The Faculty Adviser/Course Coordinator shall explain to the students the expected quality of an assignment in terms of its structure, content, presentation, etc.

**c)** Evaluated assignments will be returned to the students.

## 5. Internal Tests:

**a)** For each course there shall be two internal tests during a semester. The tentative dates of the internal tests shall be announced at the beginning of each semester.

**b)** Marks for the internal tests shall be awarded on the basis of the better of the marks scored forthe two tests for each course.

**c)** It is mandatory that all students must appear for both tests. Course Coordinators may be approached for retest if a student is unable to be present for a test due to genuine reasons. Priorpermission has to be obtained from the Head of the Department if a student is absent for a test andif he/she wishes a retest. The scheme and question paper pattern for the test papers, as well asfor the End Semester Examination, will be prepared by the Boards of Studies.

**d)** Valued answer scripts will be made available to the students for perusal within 10 workingdays from the end of the tests.

## 6. Seminar

**a)** Each student shall present a seminar in each course in each semester on a topic/area allotted/approved by the Course Coordinator.

**b)** Seminar presentations shall be done for the entire class so as to benefit all the students. The interaction of the entire class is expected during the seminar presentations.

**c)** Seminars shall be evaluated on the basis of the quality of the presentation, teaching aptitude, content, interaction etc. The evaluation shall be done by the concerned Course Coordinator.

## 7. Project Evaluation

Every student shall prepare and submit an individual Dissertation/Project in partial fulfillment of the requirements for the award of the M. Sc. Degree. The topics for the Dissertation/Project shall be approved by the Department Level Monitoring Committee (DLMC) and will be allotted to the student latest by the beginning of the Third Semester of the programme. Dissertation/Project shall be submitted at the end of the Fourth Semester, which will be evaluated by a Board of Examiners constituted and appointed by the Controller of Examinations. The maximum marks for the Dissertation/Project shall be 100, of which 20 marks shall be allotted to the viva-voce examination, which shall be conducted along with the comprehensive viva-voce examination.

## 8. Scheme of Question Paper (QP) - ESE of Theory Courses

**a)** The ESE QP of the theory courses shall be as per the following scheme:

| Type of Question | Question Numbers | No. of questions to be answered | Marks |
|---|---|---|---|
| Short Answer | 1-8 | 5 | $5 \times 3 = 15$ |
| Long Answer/Essay | 9-13 | 5 | $5 \times 12 = 60$ |
| | | **Total** | **75 marks** |

**b)** In the Long Answer/Essay part, each of the Questions 9-13 shall be from different units/modules. In each question, there shall be two choices A or B and students have to write one of them.

# M.Sc. Mathematics Course Structure and Mark Distribution

| Semester | Paper Code | Title of Paper | Instructional Hours per week | Duration of ESE | Maximum Marks | | |
|---|---|---|---|---|---|---|---|
| | | | | | CE | ESE | Total |
| I | APMM121 | Linear Algebra | 5 | 3 hours | 25 | 75 | 100 |
| | APMM122 | Real Analysis | 5 | 3 hours | 25 | 75 | 100 |
| | APMM123 | Ordinary Differential Equations | 5 | 3 hours | 25 | 75 | 100 |
| | APMM124 | Basic Topology | 5 | 3 hours | 25 | 75 | 100 |
| | APMM 12PI | Scientific Programming with Python | 5 | 3 hours | 25 | 75 | 100 |
| II | APMM221 | Abstract Algebra | 5 | 3 hours | 25 | 75 | 100 |
| | APMM222 | Measure Theory and Integration | 5 | 3 hours | 25 | 75 | 100 |
| | APMM223 | Partial Differential Equations | 5 | 3 hours | 25 | 75 | 100 |
| | APMM224 | Advanced Topology | 5 | 3 hours | 25 | 75 | 100 |
| | APMM 225 | Number Theory and Cryptography | 5 | 3 hours | 25 | 75 | 100 |
| III | APMM321 | Functional Analysis | 6 | 3 hours | 25 | 75 | 100 |
| | APMM322 | Complex Analysis | 7 | 3 hours | 25 | 75 | 100 |
| | APMM323 | Elective - I (Linear Programming Techniques) | 6 | 3 hours | 25 | 75 | 100 |
| | APMM324 | Elective - II (Advanced Topics in Graph Theory) | 6 | 3 hours | 25 | 75 | 100 |
| IV | APMM421 | Theory of Linear Operators | 6 | 3 hours | 25 | 75 | 100 |
| | APMM422 | Advanced Complex Analysis | 7 | 3 hours | 25 | 75 | 100 |
| | APMM423 | Elective - III (Statistical Methods and Nonlinear Optimization) | 6 | 3 hours | 25 | 75 | 100 |
| | APMM424 | Elective - IV (Calculus of Variations and Integral Equations) | 6 | 3 hours | 25 | 75 | 100 |
| | APMMD | Dissertation / Project | | | | 80+20 | 100 |
| | APMMV | Comprehensive Viva | | | | | 100 |
| **Grand Total** | | | | | **2000** | | |

# List of Elective Courses

## Elective Course-1

| Semester | Course Code | Course Title |
|---|---|---|
| III | APMM 323 | Automata Theory |
| III | APMM 323 | Linear Programming Techniques |
| III | APMM 323 | Difference Equations |
| III | APMM 323 | Approximation Theory |

## Elective Course - II

| Semester | Course Code | Course Title |
|---|---|---|
| III | APMM 324 | Differential Geometry |
| III | APMM 324 | Advanced Topics in Graph theory |
| III | APMM 324 | Mechanics |
| III | APMM 324 | Theory of Wavelets |

## Elective Course - III

| Semester | Course Code | Course Title |
|---|---|---|
| IV | APMM 423 | Coding Theory |
| IV | APMM 423 | Spectral Graph Theory |
| IV | APMM 423 | Statistical Methods and Nonlinear Optimization |
| IV | APMM 423 | Category Theory |

## Elective Course - IV

| Semester | Course Code | Course Title |
|---|---|---|
| IV | APMM 424 | Artificial Neural Networks and Machine Learning |
| IV | APMM 424 | Calculus of Variations and Integral Equations |
| IV | APMM 424 | Commutative Algebra |
| IV | APMM 424 | Representation theory of Finite Groups |

# PROGRAMME SPECIFIC OUTCOMES (PSO)

At the end of the programme, the student will be able to,

| | |
|---|---|
| **PSO 1** | Use knowledge of Mathematics, in all the fields of learning including higher research. |
| **PSO 2** | Analyze and solve complex mathematical problems using the knowledge of pure and applied mathematics. |
| **PSO 3** | Show their ideas by doing quality projects, discussions and participative learning. |
| **PSO 4** | Model the real-world problems in to mathematical equations and draw the inferences by finding appropriate solutions. |
| **PSO 5** | Employ confidently the knowledge of mathematical software and tools for treating the complex mathematical problems and scientific investigations. |
| **PSO 6** | Explain the contemporary issues in the field of Mathematics and applied sciences. |
| **PSO 7** | Practice mathematical knowledge and skills appropriate to professional activities keeping highest standards of ethical issues in mathematics. |
| **PSO 8** | Recognize the demands of the growing field of Mathematics by lifelong learning. |
| **PSO 9** | Apply the knowledge to qualify national level tests like NET/GATE etc. |
| **PSO 10** | Demonstrate effectively as an individual, and also as a member or leader in multi-linguistic and multi-disciplinary teams. |

# SEMESTER I

# APMM 121: Linear Algebra

**Text:**

M. Thamban Nair and Arindama Singh, *Linear Algebra*, Springer, 2018.

**COURSE OUTCOMES (CO):**

Upon the completion of this course, the students will be able to

| No | Outcome | CO - PSO Mapping |
|---|---|---|
| **CO 1** | Write the basis of a vector space, compute its dimension, set up sums of subspaces etc. | PSO 1 |
| **CO 2** | Practice various row reduction techniques of matrices in order to solve systems of linear equations. | PSO 9 |
| **CO 3** | Describe rank and nullity of a linear transformation in connection with solution of linear equations. | PSO 9 |
| **CO 4** | Employ different methods to compute eigen values and eigen vectors, which will be very useful for higher studies and research. | PSO 7, PSO 9 |
| **CO 5** | Summarize different types of representations of linear transformations along with their uses. | PSO 2, PSO 3 |

## UNIT 1

**Vector spaces:** Vector space, subspaces, linear span, linear independence, Basis and dimension, Basis of any vector space, Sums of subspaces, Quotient space. (Sections 1.1 to 1.9).

## UNIT II

**Linear transformations:** Linearity, Rank and nullity, Isomorphisms, Matrix Representation, change of basis, Space of linear transformations (Sections 2.1 to 2.7).

## UNIT III

**Elementary Operations:** Elementary row operations, Row echelon form, Row reduced echelon form, Reduction to rank echelon form, Determinant, linear equations, Gaussian and Gaussian Jordan elimination. (Sections 3.1 to 3.8).

## UNIT IV

**Eigenvalues and Eigenvectors:** Existence of eigenvalues, Characteristic polynomial, Eigenspace, Generalized eigenvectors, Two annihilating polynomials. (Sections 5.1 to 5.6).

## UNIT V

**Block diagonal representation:** Diagonalizability, Triangularizability and Block-diagonalization, Schur Triangularizations, Jordan block, Jordan Normal form. (Sections 6.1 to 6.6).

**References:**


1. David Poole, *Linear Algebra: A Modern Introduction* (IV ed.), Cengage Learning, 2015.
2. Gilbert Strang, *Linear Algebra and its Applications* (IV ed.), Cengage Learning (RS), 2005.
3. Kenneth Hoffman, Ray Kunze, *Linear Algebra* (II ed.), Prentice Hall India Learning Private Limited, 1978.
4. Peter Petersen, *Linear Algebra*, Springer, 2012.
5. Sheldon Axler, *Linear Algebra Done Right* (III ed.), Springer Nature, 2015.
6. S. Kumaresan, *Linear Algebra: A Geometric Approach*, Prentice Hall India Learning PrivateLimited, 2000.

# APMM 122: Real Analysis

**Texts:**

1. Tom M. Apostol, *Mathematical Analysis*, Second Edition, Narosa, 1974.
2. Sudhir R. Ghorpade and Balmohan V Limaye, *A course in Multivariate Calculus and Analysis*, Springer, 2010.

**COURSE OUTCOMES (CO):**

Upon completion of this course, the student will be able to

| No | Outcome | CO – PSO Mapping |
|---|---|---|
| **CO 1** | Classify and explain the notions of limit points, convergent and Cauchy sequences, continuity, connectedness and compactness, monotonic functions and bounded variations and to derive the proofs related to these concepts. | PSO 1, PSO 9 |
| **CO 2** | Summarize how completeness, continuity and other notions are generalizedfrom the real line to metric spaces. | PSO 1, PSO 2, PSO 3 |
| **CO 3** | Employ the Riemann - Stieltjes integrability condition of a boundedfunction and prove a selected number of theorems and concerning integration. | PSO 1, PSO 2 |
| **CO 4** | Recognize the differences between point wise and uniform convergence ofa sequence of functions. | PSO 9 |
| **CO 5** | Calculate the limits of two variable functions, and get the idea that, theexistence of partial derivatives will not imply the differentiability of the function as in single variable case. | PSO 9 |
| **CO 6** | Describe how to use implicit function theorem. | PSO 8 |

**UNIT I**

**Review of the following topics:** Continuity - Limits of function, Continuous functions, Continuity and compactness, continuity and connectedness, Discontinuities, Monotonic functions, Infinite limits and Limits at Infinity.

**Functions of Bounded Variation and Rectifiable Curves:** Properties of monotonic functions, Functions of bounded variation, Total variation, Additive property of total variation, Total variation on [a, x] as an increasing function, Function of bounded variation expressed as the difference of increasing functions, Continuous functions of bounded variation, Curves and paths, Rectifiable paths and arc length, Additivity and continuity of arc length, Equivalence of paths, Change of parameter.
[Chapter 6 of Text 1].

## UNIT II

**The Riemann - Stieltjles Integral:** The definition of Riemann - Steiltjles integral, Linear properties, Integration by parts, Change of variable in a Riemann - Stieltjes integral, Reduction to a Riemann integral, Step functions as integrators, Reduction of a Riemann - Stieltjes integral to a finite sum, Euler's summation formula, Monotonically increasing integrators, Upper and lower integrals, Additive and linear properties of upper and lower integrals, Riemann's condition, Comparison Theorems, Integrators of bounded variation, Sufficient conditions for the existence of Riemann - Stieltjes integrals, Differentiation under the integral sign.
[Chapter 7: Sections 7.1 - 7.16 and 7.24 of Text 1].

## UNIT III

**Sequence of Functions:** Point-wise convergence of sequences of functions, Examples of sequences of real-valued functions, Definition of uniform convergence, Uniform convergence and continuity, the Cauchy condition for uniform convergence, Uniform convergence of infinite series of functions, Uniform convergence and Riemann - Stieltjes integration, Non-uniformly convergent series that can be integrated term by term, uniform convergence and differentiation, sufficient conditions for uniform convergence of a series. (Do more problems to study the uniform convergence of sequences and series).
[Chapter 9: Sections 9.1- 9.9 (excluding Section 9.7) of Text 1].

## UNIT IV

**Multivariate Calculus, Sequences, Continuity and Limits:** Sequences in $\mathbf{R}^2$, Sub-sequences and Cauchy sequences, Compositions of continues functions, Piecing continuous functions on overlapping subsets, Characterizations of continuity, Continuity and boundedness, Continuity and convexity, Continuity and intermediate value property, Uniform continuity, Implicit function Theorem, Limits and continuity.
[Chapter 2: Sections 2.1, 2.2 (excluding Continuity and monotonicity, Continuity, Bounded Variation, Bounded Bivariation), 2.3 (Excluding Limits from a quadrant, Approaching Infinity) of Text 2].

## UNIT V

**Partial and Total Differentiation:** Partial derivative, Directional derivatives, Higher order partial derivatives, Higher order directional derivatives, Differentiability, Taylor's Theorem and Chain rule, Functions of three variables, Extensions and analogues, Tangent planes normal lines to surfaces. [Chapter 3 (excluding Section 3.4 and last subsection of Section 3.5) of Text 2].

## References:

1. J. A. Dieudonne, *Foundations of Modern Analysis*, Academic Press, 1960.
2. W. Rudin, *Real and Complex analysis*, Third Edition, Tata Mc-Graw Hill, 1987.
3. Tom M. Apostol, *Calculus,* Volume-1, Second Edition, Wiley, 1991.
4. Tom M. Apostol, Calculus, Volume-2, Second Edition, Wiley, 1975.
5. N. L. Carothers, *Real Analysis*, Cambridge University Press, 2000.

# APMM 123: Ordinary Differential Equations

## Texts:

George F. Simmons, *Differential Equations with Applications and Historical Notes*, 3rd edition,CRC Press, Taylor and Francis Group, 2016.

## COURSE OUTCOMES (CO):

Upon completion of this course, the student will be able to

| No | Outcome | CO – PSO Mapping |
|---|---|---|
| **CO1** | Apply Picard's Theorem to check the existence of solution of differential equations. | PSO 1, PSO 2, PSO 3 PSO 9 |
| **CO2** | Classify the singular points of second order differential equations with variable coefficients. | PSO 7, PSO 9 |
| **CO3** | Write power series solutions of several important classes of ordinary differential equations including Bessel's, Legendre, Gauss's hypergeometric differential equations. | PSO 9 |
| **CO4** | Analyse the stability of linear and non-linear system of differential equations. | PSO 4 |
| **CO5** | Summarize the oscillation properties of differential equations. | PSO 4 |

## UNIT I

**Existence and Uniqueness of Solutions:** The method of successive approximations, Picard's theorem, systems. The second order linear equation, Oscillations and Sturm Separation theorem, Sturm Comparison theorem. [Chapter 13: Sections 69, 70, 71; Chapter 4: Sections 24, 25].

## UNIT II

**Power Series solutions:** A review of power series, series solutions of first order equations, second order linear equations, ordinary points, regular singular points, two convergence proofs. Gauss's Hypergeometric Equation, The Point at Infinity. [Chapter 5: Sections 26, 27, 28, 29, 30, 31, 32 and Appendix A].

## UNIT III

**Special functions:** Legendre Polynomials and their properties, Bessel Functions, the Gamma Function, Properties of Bessel Functions, Additional Properties of Bessel Functions. [Chapter 8: Sections 44, 45, 46, 47 and Appendix C)].

## UNIT IV

**System of first order equations:** General remarks on systems, linear systems, Homogenous linear systems with constant coefficients, non-linear systems. Volterra's Prey-Predator equations. [Chapter 10:Sections 54, 55, 56, 57].

## UNIT V

**Non-linear equations:** Autonomous systems. The Phase Plane and Its Phenomena, Types of Critical Points. Stability, Critical points and Stability for Linear Systems, Simple critical points of non-linear systems. [Chapter 16: Sections 58, 59, 60, 62].

## References:


1. E. A. Coddington & N. Levinson, *Theory of Ordinary Differential Equations*, Tata-McGraw Hill,2012.
2. W. Walter, *Ordinary Differential Equations*, Springer, 6th edition, 1996.
3. P. Blanchard, R. L. Devaney & G. R. Hall, *Differential Equations*, Brooks/Cole, 3rd edition, 2006.
4. G. F. Simmons, *Differential Equations with Applications and Historical Notes*, McGraw Hill, 2nd edition, 1991.
5. Dennis G. Zill, *A First Course in Differential Equations with Modeling Applications*, Brooks/Cole, 6th edition, 1997.
6. I. Sneddon, *Elements of Partial Differential Equations*, Dover Publications, Inc., 2006.
7. Erwin Kreyszig, *Advanced Engineering Mathematics*, 9th edition, John Wiley and Sons, 2011.
8. D. Greenspan, *Introduction to Partial differential Equations*, TMH Edition, 1961.
9. K. Sankara Rao, *Introduction to Partial Differential Equations*, 3rd edition, PHI Learning, 2011.

# APMM 124: Basic Topology

**Text:**

Principles of Topology, *Fred H. Croom*, Baba Barkha Nath Printers (India), Third Reprint, 2009.

**COURSE OUTCOMES (CO):**

Upon completion of this course, the student will be able to

| No | Outcome | CO - PSO Mapping |
|---|---|---|
| **CO 1** | Construct topological spaces from metric spaces and using general properties of neighborhoods, open sets, closed sets, basis and sub basis. | PSO 1, PSO 2 |
| **CO2** | Apply the properties of open sets, closed sets, interior points, accumulation points and derived sets in deriving the proofs of various theorems. | PSO 3, PSO 9 |
| **CO3** | Interpret and apply the concepts of countable spaces and separable spaces. | PSO 7 |
| **CO4** | Demonstrate compactification and related theorems. | PSO 2 |
| **CO5** | Justify the concepts and properties of the compact and connected topological spaces. | PSO 9 |

**UNIT I**

**Metric Spaces:** Definition, Examples, Open Sets, Closed Sets, Interior, Closure and Boundary. [Sections: 3.1, 3.2 and 3.3].

**UNIT II**

**Continuous Functions:** Equivalence of metric spaces, Complete metric spaces, Cantor's IntersectionTheorem. [Sections: 3.4, 3.5 and 3.7 (Exercise may be included 3.7(3)].

**UNIT III**

**Topological Spaces:** Definition, Examples, Interior, Closure, Boundary, Base, Sub base, Continuity, Topological Equivalence, Subspaces. [Sections: 4.1, 4.2, 4.3, 4.4 and 4.5].

**UNIT IV**

**Connectedness and disconnected spaces:** Theorems on connectedness, connected subsets of realline, Applications of Connectedness, Path connected spaces. [Sections: 5.1, 5.2, 5.3, 5.4 and 5.5].

**UNIT V**

**Compact spaces:** compactness and continuity, properties related to compactness, one pointcompactification. [Sections: 6.1, 6.2, 6.3 and 6.4].

**References:**


1. Gerald Buskes, Arnoud van Rooiji, *Topological Spaces from Distance to Neighbourhood*, Springer, First Edition, 1997.
2. James R. Munkres, *Topology*, PHI Learning Private Limited, Second Edition, 2009.
3. Stephen Willard, *General Topology*, Dover Publications, 1970.
4. G. F. Simmons, *Topology and Modern Analysis*, Mc Graw-Hill Inc, New York, 13th reprint, 2010.
5. J. Arthur Seebach, Lynn Arthur Steen, *Counter Examples in Topology*, Dover Publications, 1995.
6. Sheldon W. Davis, *Topology*, Tata Mc Graw-Hill, 2006.

# APMM 12PI: Scientific Programming with Python

**Texts:**

1. Vernon L. Ceder, *The Quick Python Book*, Second Edition, Manning, 2010.
2. Jaan Kiusalaas, *Numerical Methods in Engineering with Python3*, Cambridge University Press, 2013.
3. Amit Saha, *Doing Math with Python*, No Starch Press, 2015.

## COURSE OUTCOMES (CO):

The course is intended as a basic course in numerical analysis using Python. The objective of the course is to familiarize the students about the language Python and to learn different numerical techniques like solving algebraic and transcendental equations, large linear system of equations, differential equations, approximating functions by polynomials up to a given desired accuracy, finding approximate value of definite integrals of functions etc.

Upon completion of the course, the students should be able to

| No. | Outcomes | CO - PSO Mapping |
|---|---|---|
| **CO 1** | Recognize the language of Python and will be trained to write some codes for doing computations and also analyze the efficiency of any numerical algorithm. | PSO 4, PSO 5 |
| **CO 2** | Calculate numerical solution of nonlinear equations using Bisection, Secant, Newton and Fixed - Point iterations methods and convergence analysis of these methods. | PSO 6 |
| **CO 3** | Solve linear systems of equations numerically. | PSO 10 |
| **CO 4** | Apply numerical methods to handle the functions and data set using interpolation and least square curves. | PSO 8 |
| **CO 5** | Employ the techniques to evaluate the integrals numerically. | PSO 4, PSO 8 |

## UNIT I

In this unit we discuss the basics of python. The topics are based on chapters 4, 5, 6, 7, 8, 9, and 10 of Text 1. All topics of chapters 4-9 must be discussed using examples from mathematics. In chapter 10,only sections 10.1-10.4 need to be discussed. We also discuss the basics of *numpy* package based on sections 3.1 to 3.23 of reference 1. The students should be encouraged to write programs related with mathematical problems. (Some of the problems are listed in the syllabus).

## UNIT II

Visualizing Data with Graphs - learn a powerful way to present numerical data: by drawing graphs with Python. The unit is based on Chapter 2 of Text 3. The sections Creating Graphs with Matplotliband Plotting with Formulas must be done in full. In the section Programming Challenges, the problems Exploring a Quadratic Function Visually, Visualizing Your Expenses and Exploring the Relationship between the Fibonacci Sequence and the Golden Ratio must also be discussed.

## UNIT III

The unit is based on chapters 4 and 7 of Text 3. Here we discuss Algebra and Symbolic Math with SymPy and Solving Calculus Problems. In Chapter 4 the sections Defining Symbols and Symbolic Operations, Working with Expressions, Solving Equations and Plotting Using SymPy should be done in full. In the section Programming Challenges, the problems Factor inder, Graphical Equation Solver, summing a Series and Solving Single-Variable Inequalities also should be discussed. In chapter 7, some problems are discussed namely, Finding the Limit of Functions, Finding the Derivative of Functions, Higher-Order Derivatives and Finding the Maxima and Minima and Finding the Integrals of Functions are to be done. In the section Programming Challenges, the problems Verify the Continuity of a Function at a Point, Area Between Two Curves and Finding the Length of a Curve also should be discussed.

## UNIT IV

In units IV and V we discuss some numerical methods and the corresponding python programs. The emphasis is given for programming rather than the methods. The methods include the following.

Gauss Elimination Method (excluding Multiple Sets of Equations), LU Decomposition Methods (Doolittle's Decomposition Method only). [Sections 2.2, 2.3 of Text 2].

Interpolation and Curve Fitting - Polynomial Interpolation - Lagrange's Method, Newton's Method and Limitations of Polynomial Interpolation. [Sections 3.1 and 3.2 of Text 2].

Roots of Equations - Method of Bisection and Newton-Raphson Method. [Sections 4.1, 4.3 and 4.5 of Text 2].

## UNIT V

Numerical Integration - Newton-Cotes Formulas - Trapezoidal rule, Simpson's rule and Simpson's 3/8rule. [Sections 6.1 and 6.2 of Text 2].

Initial Value Problems - Euler's Method and Runge - Kutta Methods. [Sections 7.1, 7.2 and 7.3 of Text2].

Some problems for Unit I are listed below:

- Checking primality of a number.
- Listing all primes below a given number.
- Prime factorization of a number.
- Finding all factors of a number.
- gcd of two numbers using the Euclidean Algorithm.
- Finding the multiples in Bezout's Identity.
- checking the convergence and divergence of sequences and series.
- Matrix multiplication.
- Matrix inversion.

(For more problems visit : https://www.nostarch.com/doingmathwithpython/, https://doingmathwithpython.github.io/author/amit-saha.html and https://projecteuler.net/.)

- The course is aimed to give an introduction to mathematical computing, with Python as tool for computation.
- The students should be encouraged to write programs to solve the problems given in thesections as well as in the exercises.
- The end semester evaluation is only a practical examination.
- Practical examination will be of 3 hours duration for a maximum of 75 marks.
- One question out of 2 from each unit should be answered. The questions will carry equal marks.
- The practice of writing the record should be maintained by each student throughout the course and it should be dually certified by the teacher in charge/internal examiner and evaluated by the external examiner of practical examination.

**References:**


1. *NumPy User Guide Release 1.20.0,* Written by the NumPy community. (available athttps://numpy.org/doc/1.20/numpy-user.pdf).
2. https://docs.python.org/3/tutorial/
3. Richard L. Burden and J. Douglas Faires, *Numerical Analysis*, Ninth Edition, Brooks/Cole, Cengage Learning, 2011.

# SEMESTER II

# APMM 221: Abstract Algebra

### Text:

David S. Dummit and Richard M. Foote, *Abstract Algebra*, 3rd Edition, Wiley Publications, 2003.

Students are introduced to some basic concepts and theorems of groups, rings, integral domains, etc. during their previous courses in the undergraduate programme. In this master's level course, advancedtopics in group, ring, and field theory are discussed.

### COURSE OUTCOMES:

This course is meant basically to help the students to have an advanced mastery of group, ring, module and field theories. A solid foundation in the theories of these topics will help the students in the application of these concepts in various branches of Science.

After the completion of the course, the students will be able to

| No | Outcome | CO – PSO Mapping |
|---|---|---|
| **CO 1** | Describe the concept of group actions culminating in Sylow's Theorems and the simplicity of $A_n$. | PSO 2, PSO 3 |
| **CO 2** | Explain the direct and semi direct product of groups and the fundamental theorem of finitely generated abelian groups with the help of which being able to classify groups of small orders. | PSO 9 |
| **CO 3** | Express the basics of field extensions and splitting fields. | PSO 9 |
| **CO4** | Apply the fundamental theorem of Galois Theory with the help of which being able to prove the insolvability of the quintic and to form Galois groups. | PSO 2, PSO 9 |
| **CO5** | Summarize the basics of module theory and tensor products of modules and modules over Principal Ideal Domains. | PSO 10 |

### UNIT 1

**Group Actions:** Group actions and permutation representations, Groups acting on themselves by left multiplication - Cayley's Theorem, Groups acting on themselves by conjugation - The Class Equation, Automorphisms, The Sylow Theorems, The simplicity of $A_n$. (Sections 4.1 to 4.6 of the Text book).

### UNIT II

**Direct and Semi direct products and Abelian groups:** Direct Products, the fundamental theorem of finitely generated abelian groups, Table of groups of small order, Recognizing direct products, Semidirect products, p-groups, nilpotent groups and solvable groups. (Sections 5.1 to 5.5 and 6.1 of the Text book).

## UNIT III

**Introduction to Module Theory:** Basic Definitions and examples, Quotient modules and module homomorphisms, Generation of modules, direct sums and free modules, Tensor products of modules Exact sequences, projective, injective and flat modules, Modules over Principal Ideal domains, the basic theory. (Sections 10.1 to 10.5 and 12 of the Text book).

## UNIT IV

**Field Theory:** Basic theory of field extensions, Algebraic extensions, Classical straight edge and compass constructions, Splitting fields and algebraic closures, Separable and inseparable extensions, Cyclotomic polynomials and extensions. (Sections 13.1 to 13.6 of the Text book).

## UNIT V

**Galois Theory:** Basic definitions, The fundamental theorem of Galois Theory, Finite fields, Composite extensions and simple extensions, Cyclotomic extensions and abelian extensions over Q, Galois groups of polynomials, Solvable and radical extensions, insolvability of the quintic, Computation of Galois groups over Q, Transcendental extensions, inseparable extensions, infinite Galois groups. (Theorem 14 may be discussed without proof) (Sections 14.1 to 14.9 of the Text book).

## References:


1. Joseph A. Gallian, *Contemporary Abstract algebra*, 8$^{th}$ Edition, Brooks/Cole, Cengage Learning, 2010.
2. John B. Fraleigh, *A first course in abstract algebra*, 7$^{th}$ edition, Pearson Education Inc, 2003.
3. T. W. Hungerford, *Algebra*, Springer, 2005.
4. I. N. Heirstein, *Topics in Algebra*, John Wiley & Sons, 1975.
5. M. Artin, *Algebra*, Prentice Hall, 1991.

# APMM 222: Measure Theory and Integration

**Text:**

G de Barra, *Measure Theory and Integration*, New Age International Publishers, New Delhi, 1981.

## COURSE OUTCOMES (CO)

This course is intended to give a mathematical foundation to probability theory and statistics and to covey the idea that, on the real line it gives a natural extension of the Riemann integral which allows for better understanding of the fundamental relations between differentiation and integration.

After the successful completion of this course, the students will be able to

| No | Outcome | CO - PSO Mapping |
|---|---|---|
| **CO 1** | Give Examples how Lebesgue measure on $\mathbb{R}$ is defined. | PSO 9 |
| **CO 2** | Extend how measures may be used to construct integrals. | PSO 1 |
| **CO 3** | Relate the basic convergence theorems for the Lebesgue integral. | PSO 2, PSO 9 |
| **CO 4** | Explain the concept of Spaces and its properties. | PSO 6 |
| **CO 5** | Apply the concept of measure to signed measure and interpret a selection of theorems concerning signed measure and Radon Nikodym derivative. | PSO 8 |

## UNIT I

**Lebesgue Outer Measure:** Measurable sets, Regularity, Measurable functions, Borel and Lebesgue Measurability. (Chapter 2: Sections 1 - 5).

## UNIT II

**Integration of Non-negative functions:** The General Integral, Integration of Series, Riemann and Lebesgue Integrals, The Four Derivatives, Lebesgue's Differentiation Theorem, Differentiation and Integration. (Chapter 3: Sections 1 - 4; Chapter 4: Sections 1, 4 – statements only and 5).

## UNIT III

**Abstract Measure Spaces:** Measures and Outer Measures, Extension of a measure, Uniqueness of the Extension, Completion of the Measure, Measure spaces, Integration with respect to a Measure. (Chapter5: Sections 1 - 6).

## UNIT IV

**The Spaces:** Convex Functions, Jensen's Inequality, The Inequalities of Holder and Minkowski, Completeness of $L^p(\mu)$. (Chapter 6: Sections 1 - 5).

## UNIT V

**Convergence in Measure:** Signed Measures and the Hahn Decomposition, The Jordan Decomposition, The Radon Nikodym Theorem, Some Applications of the Radon Nikodym Theorem. (Chapter 7: Section1; Chapter 8: Sections 1 - 4).

**References:**


1. M. Thamban Nair, *Measure and Integration*, *A first course*, CRC Publishers, 2019.
2. H. L. Roydon, *Real Analysis*, Third Edition, Mac – Millan, 1988.
3. W. Rudin, *Principles of Mathematical Analysis*, 3rd Edition, 1964.
4. P. R. Halmos, *Measure Theory*, Springer, 1950.
5. E. M. Stein & R. Shakarchi, *Real analysis: Measure Theory*, *Integration, and Hilbert Spaces*, Princeton University Press, 2005.
6. G. B. Folland, *Real analysis:Modern Techniques and Their Applications*, John Wiley & Sons, 2nd edition, 1999.

# APMM 223: Partial Differential Equations

## Text:

Yehuda Pinchover and Jacob Rubinstein, *An Introduction to Partial Differential Equations*, Cambridge University Press, 2005.

## COURSE OUTCOMES:

Upon the completion of this course, the student will be able to

| No | Outcome | CO – PSO Mapping |
|---|---|---|
| **CO 1** | Employ the method of characteristics, and apply existence and uniqueness theorem. | PSO 2, PSO 9 |
| **CO 2** | Analyze different types of second order partial differential equations along with their canonical forms. | PSO 4, PSO 9 |
| **CO 3** | Solve a PDE using the method of separation of variables. | PSO 9 |
| **CO 4** | Relate the beauty of Inner product spaces. | PSO 8 |
| **CO 5** | Classify the second order partial differential equations and use the method of separation of variables. | PSO 9 |
| **CO 6** | Interpret the physics and hence the applications behind all the standard second order partial differential equations. | PSO 4, PSO6, PSO 8 |

## UNIT I

**First Order equations:** Introduction, quasi linear equations, the method of characteristics, examples of the characteristic method, the existence and uniqueness theorem, the Lagrange method, Conservation laws and shock waves, the eikonal equation, general nonlinear equations. [Chapter 2: Sections 2.1 – 2.9].

## UNIT II

**Second Order linear equations in two independent variables:** Introduction, classification, canonical form of hyperbolic equations, canonical form of parabolic equations, canonical form of elliptic equations, Introduction to one dimensional wave equation, canonical form and general solution, the Cauchy problem and d'Alembert's formula. [Chapter 3: Sections 3.1 - 3.5; Chapter 4: Sections 4.1 - 4.3].

## UNIT III

**Method of separation of variables:** Domain of dependence and region of influence, the Cauchy problem for the nonhomogeneous wave equation, introduction to the method of separation of variables, Heat equation: homogeneous boundary condition, separation of variables for the wave equation, separation of variables for the nonhomogeneous equations. [Chapter 4: Sections 4.4, 4.5, Chapter 5: Sections 5.1 – 5.4].

## UNIT IV

**Sturm – Liouville problems and eigen function expansions:** Introduction, the Sturm-Liouville problem, inner product spaces and orthonormal systems, basic properties of Sturm – Liouville eigen functions and eigenvalues, nonhomogeneous equations, nonhomogeneous boundary conditions. [Chapter 6: Sections 6.1 – 6.6].

## UNIT V

**Equations in high dimensions:** Introduction, first order equations, classification of second order equations, the wave equation in $R^2$ and $R^3$, the eigenvalue problem for the Laplace equation, separation of variables for the heat equation, separation of variables for the wave equation, separation of variables for the Laplace equation. [Chapter 9: Sections 9.1 – 9.8].

## References:


1. I. Sneddon, *Elements of Partial Differential Equations*, Dover Publications, Inc., 2006.
2. D. Greenspan, *Introduction to Partial Differential Equations*, TMH Edition, 1961.
3. Erwin Kreyszig, *Advanced Engineering Mathematics*, John Wiley and Sons, 1995.
4. K. S. Rao, *Introduction to Partial Differential Equations*, PHI Learning Pvt. Ltd., 2011.
5. K. S. Rao, *Introduction to Partial Differential Equations*, PHI Learning Pvt. Ltd., 2011.

# APMM 224: Advanced Topology

## Texts:

1. Fred H Croom, *Principles of Topology*, Baba Barkha Nath Printers (India), Third Edition, reprint 2009.

2. Sheldon W. Davis, *Topology*, Tata Mc Graw-Hill Edition, 2006.

## COURSE OUTCOMES (CO):

After the completion of this course, the students will be able to

| No | Outcome | CO - PSO Mapping |
|---|---|---|
| **CO 1** | Construct product and quotient topological spaces and compare different topologies defined on the same underlying set. | PSO 1, PSO 3 |
| **CO 2** | Interpret the separation axioms and analyze their different characterizations based on a geometrical manner. | PSO 8, PSO 10 |
| **CO 3** | Give examples to recognize the idea on separation by continuous functions and on embedding theorems. | PSO 1 |
| **CO 4** | Illustrate the fundamentals of algebraic topology. | PSO 8 |
| **CO 5** | Recognize an entry into fixed point theory in a topological view. | PSO 3 |

## UNIT I

**Product and Quotient Spaces:** Finite and arbitrary products, Comparison of topologies, Quotient spaces. [Chapter 7: Sections 1 - 4 of Text - 1 (excluding Alexander sub basis theorem and Theorem- 7.11)].

## UNIT II

**Separation axioms:** *T0*, *T1* and *T2* – spaces, Regular spaces, Normal spaces, Separation by continuousfunctions. [Chapter 8: Sections 1 - 4 of Text - 1].

## UNIT III

**Convergence:** Tychonoff's Theorem. [Chapter 16: Theorem 18.21 and Theorem 18.22 of Text - 2].

## UNIT IV

**Algebraic Topology:** The fundamental group, The fundamental group of $S^1$. [Chapter 9: Sections 1- 3 of Text - 1].

## UNIT V

**Fixed point Theorem:** Examples of fundamental groups, The Brouwer Fixed Point Theorem. [Chapter 9: Sections 4-5 of Text - 1].

**References:**


1. Gerald Buskes, Arnoud van Rooiji, *Topological Spaces from Distance to Neighbourhood*, Springer, First Edition, 1997.
2. James R. Munkres, *Topology*, PHI Learning Private Limited, Second Edition, 2009.
3. Stephen Willard, *General Topology*, Addison-Wesley, Reading, 1970.
4. G. F. Simmons, *Topology and Modern Analysis*, Mc Graw-Hill Inc, New York, $13^{th}$ reprint, 2010.
5. J. Arthur Seebach, Lynn Arthur Steen, *Counter Examples in Topology*, Dover Publications,1995.
6. Sheldon W. Davis *Topology*, Tata Mc Graw-Hill, 2006.

# APMM 225: Number Theory and Cryptography

**Text:**

Neal Koblitz, *A Course in Number Theory and Cryptography*, 2nd edition, Springer Verlag, 1994.

**COURSE OUTCOMES:**

Upon the completion of this course, the student will be able to

| No | Outcome | CO – PSO Mapping |
|---|---|---|
| **CO 1** | Explain the properties of divisibility, prime numbers and arithmetic functions in different areas of mathematics and to relate with the theory of finite Abelian groups and their characters. | PSO 1, PSO 4, PSO 9 |
| **CO 2** | Describe quadratic residues and to use reciprocity law. | PSO 7 |
| **CO 3** | Recognize the existence of primitive roots. | PSO 9 |
| **CO 4** | Summarize the basics of RSA security and be able to separate the simplest instances and to apply the use of simple crypto systems. | PSO 4, PSO 5 |
| **CO 5** | Experiment with Fermat factorization and factor bases, the quadratic sieve method. | PSO 2 |

**UNIT I**

Time estimates for doing arithmetic, divisibility and the Euclidean algorithm. (Chapter – I Sections 1, 2 of the text).

**UNIT II**

Congruences, some applications to factoring, finite fields. (Chapter - I Sections 3, 4; Chapter –II Section 1 of the text).

**UNIT III**

Quadratic residues and reciprocity, the idea of public key cryptography. (Chapter - II Section 2; Chapter IV Sections 1 of the text)

**UNIT IV**

RSA**,** Discrete logarithm, Pseudo primes. (Chapter – IV sections 2, 3; Chapter – V Section 1 of the text)

**UNIT V**

The rho method, Fermat factorization and factor bases, the quadratic sieve method. (Chapter – VSections 3, 4, 5 of the text).

**References:**


1. Niven, H. S. Zuckerman and H. L. Montgomery, *An introduction to the theory of numbers*, John Wiley, 5th Edition, 1991.
2. Baldoni, M. Welleda, Ciliberto, Ciro, Piacentini Cattaneo, G. M, *Elementary NumberTheory, Cryptography and Codes*, Springer, 2009.
3. Ireland and Rosen, *A Classical Introduction to Modern Number Theory*. Springer, 2nd edition, 1990.
4. David Burton, *Elementary Number Theory and its applications*, Mc Graw-Hill Education (India) Pvt. Ltd, 2006.
5. Alfred J. Menezes, Paul C. van Oorschot and Scott A. Vanstone, *Handbook of Applied Cryptography*, CRC Press, 2001.
6. Douglas R. Stinson, *Cryptography Theory and Practice*, Chapman & Hall, 2nd edition,2019.
7. Victor Shoup, A *computation Introduction to Number Theory and Algebra*, CambridgeUniversity Press, 2005.
8. William Stallings, *Cryptography and Network Security Principles and Practice*, Third edition, Prentice - hall, India, 2005.

# SEMESTER III

# APMM 321: Functional Analysis

**Text:**

Erwin Kreyszig, *Introductory Functional Analysis with Applications*, Wiley India, 2007.

**COURSE OUTCOMES (CO):**

This course is intended to the students for a better understanding about the link between Mathematics and its applications. Functional Analysis deals with the interplay between algebraic structures and distance structures.

Upon the successful completion of this course, the students will be able to

| No | Outcome | Co – PSO Mapping |
|---|---|---|
| **CO 1** | Recognize the necessity of defining a norm and an inner product on alinear space and analyze how the structure of a linear space changes. | PSO 1, PSO 2, PSO 9 |
| **CO 2** | Explain the properties of linear maps on finite and infinite dimensional normed, inner product spaces. | PSO 6, PSO 9 |
| **CO 3** | Interpret Hahn Banach theorems and apply them to extend linear functionals on subspaces to normed spaces. | PSO 3 |
| **CO 4** | Illustrate uniform boundedness theorem, experiment with Fourier series, and thereby summarize the link between functional analysis and applications. | PSO 2, PSO 8 |
| **CO 5** | Demonstrate Open Mapping theorem and Closed Graph theorem. | PSO 2, PSO 8 |
| **CO 6** | Apply the notions of weak, weak* convergences and reflexivity. | PSO 2 |

**UNIT I**

**Normed Spaces and Banach Spaces:**

Finite and Infinite dimensional vector spaces, Hamel Basis, Normed spaces, Examples, Incomplete normed spaces and their completion, Translation invariance, Banach spaces, Subspace of a Banach space, Finite dimensional normed spaces, Equivalent norms, Compactness and finite dimension, Riesz's Lemma, Bounded and Continuous linear operators, Operator norm and its computation, linear functionals. [Sections 2.1 - 2.8 of the text].

**UNIT II**

**Inner Product spaces, Hilbert spaces:**

Definition and Examples of Inner Product spaces and Hilbert spaces, Schwarz Inequality, Triangle Inequality, continuity of inner product, orthogonal complements and direct sum, minimizing vector, orthogonality, orthogonal projection, orthonormal sets and sequences, Bessel's inequality, Series related to orthonormal sets and sequences, total orthonormal sets and sequences. [Sections: 3.1 - 3.6 of the text].

## UNIT III

**Applications of total orthonormal sequences, Some operators and Dual space:**

Legendre, Hermite and Laguerre Polynomials, Riesz's representation theorem on Hilbert spaces, Sesquilinear form, Hilbert Adjoint operator and its properties, Self-adjoint, Unitary, Normal operators, Algebraic dual space, linear operators and functionals on finite dimensional spaces, normed space of operators, some standard dual spaces. [Sections: 2.9, 2.10, 3.7 - 3.10].

## UNIT IV

**Fundamental Theorems and reflexivity:**

Application of Zorn's lemma, Hamel Basis, Hahn Banch Theorems, application to bounded linear functionals on $C[a, b]$, adjoint operator, relation between adjoint operator and Hilbert adjoint operator, Reflexive spaces, results on completeness and separability, Uniform boundedness theorem and applications, [Sections: 4.1 - 4.7],

## UNIT V

**Strong and weak convergence, Open mapping and Closed graph theorems:**

Definitions and properties of strong and weak convergence, Convergence of sequence of operators and functionals, Strong operator convergence, Application to summability of sequences, Numerical integration and weak* convergence, Open mapping theorem, closed graph theorem. [Sections: 4.8 - 4.13].

## References:


1. Bryan Rynne, M. A. Youngson, *Linear Functional Analysis*, Springer, 2008.
2. Rajendra Bhatia, *Notes on Functional Analysis*, Hindustan Book Agency, 2015.
3. M. Thamban Nair, *Functional Analysis: A First Course*, Prentice Hall of India Pvt. Ltd., 2002.
4. Walter Rudin, *Functional Analysis, 2nd Edition*, Tata Mc Graw Hill, 1995.
5. B. V. Limaye, *Linear Functional Analysis for Scientists and Engineers*, Springer Singapore, 2016.

# APMM 322: Complex Analysis

**Text:**

John B Conway, *Functions of one Complex Variables*, (2nd Edition) Springer Verlag, New York, 1973.

**COURSE OUTCOMES (CO):**

Upon completion of this course, the student will be able to

| No | Outcome | CO – PSO Mapping |
|---|---|---|
| **CO 1** | Describe the Cauchy - Riemann equations, analytic functions, entire functions. | PSO 1, PSO2, PSO 9 |
| **CO 2** | Apply power series to represent an analytic function and to Calculate the radius of convergence. | PSO 3, PS0 4, PSO 9 |
| **CO 3** | Apply the Cauchy integral theorem to evaluate complex contour integrals. | PSO 2, PSO 9 |
| **CO 4** | Classify singularities and find residues to evaluate complex integrals using the residue theorem. | PSO 2, PSO 9 |
| **CO 5** | Explain the concept of Mobius transformation, maximum principle and Schwarz's Lemma. | PSO 2, PSO 9 |

## UNIT I

**Analytic Functions:** Elementary properties and examples of analytic functions, Chain rule (statement only), Power series, Analytic functions, Riemann - Stieltjes Integrals (Lemma 1.19 statement only). [Chapter3: Sections 1, 2; Chapter 4: Section 1].

## UNIT II

**Power series representations:** Power series representation of an analytic function, zeros of an analytic function, the index of a closed curve. [Chapter 4: Sections 2, 3 and 4].

## UNIT III

**Integral Theorems:** Cauchy's theorem and integral formula, Homotopic version of Cauchy's theorem, Simple connectivity, Counting zeros, the open mapping theorem, Goursat's Theorem.
[Chapter 4: Sections 5, 6, 7 and 8 (avoid the proof of theorem 6.7)].

## UNIT IV

**Residues:** Classification of singularities, Residues (Example 2.12 only for review), the argument Principle, Analytic continuations, Monodromy theorem. [Chapter 5: Sections 1, 2 and 3; Chapter 9: Sections 2, 3].

## UNIT V

The extended plane and its spherical representation (Chapter 1: Section 6 meant for self-study).

Mobius Transformations: Mobius transformations, The Maximum Principle, Schwarz's Lemma. [Chapter 3: Section 3; Chapter 6: Sections 1, 2; Chapter 8: Sections 1, 2].

## References:

1. R. A. Silverman, *Complex Analysis with Applications*, Dover Publications, 1974.
2. L. V. Ahlfors, *Complex Analysis*, Mc-Graw Hill, 1966.
3. S. Lang, *Complex Analysis*, Mc-Graw Hill, 1998.
4. S. Ponnusamy & H. Silverman, Complex Variables with Applications, Birkhauser, 2006.
5. H. A. Priestley, *Introduction to Complex Analysis*, Oxford University Press Tristan. Needham, Visual Complex Analysis, Oxford University Press, 1999.
6. V. Karunakaran, *Complex Analysis*, Narosa Publishing House, 2005.

# APMM 323: Automata Theory (Elective - I)

**Text:**

J E Hoporoft and J D Ullman, *Introduction to Automata Theory Languages and Computation*, Narosa, 1999..

**COURSE OUTCOMES (CO):**

Upon the completion of this course on Automata theory, the students will be able to

| No | Outcome | CO – PSO Mapping |
|---|---|---|
| CO 1 | Explain the basics of languages and automata. | PSO 1, PSO 2 |
| CO 2 | Identify different types of grammars. | PSO 5, PSO 8 |
| CO 3 | Interpret the concept of Push down automata. | PSO 5, PSO 8 |
| CO 4 | Demonstrate the notion of Turing machine. | PSO 10 |
| CO5 | Apply Context free languages and Chomsky hierarchy. | PSO 10 |

**UNIT I**

**Introduction to Automata Theory:** Strings, Alphabets and Languages (Section 1.1 of the Text), FiniteAutomata. (Chapters 2: Sections 2.1 - 2.4).

**UNIT II**

**Expressions:** Regular expressions and Properties of Regular sets. (Sections 2.5 - 2.8, 3.1 - 3.4).

**UNIT III**

**Grammars:** Context Free grammars. (Sections 4.1 - 4.5).

**UNIT IV**

**Pushdown Automata:** Pushdown Automata & properties of Context free languages Theorem 5.3, 5.4(without proof). (Sections 5.1 - 5.3, 6.1 - 6.3).

**UNIT V**

**Turing Machine:** Turning Machine and Chomsky hierarchy. (Sections 7.1 - 7.3, 9.2 - 9.4).

**References:**

1. G. E. Revesz, *Introduction to Formal Languages*, Society for Industrial and Applied Mathematics,1989.
2. P. Linz, *Introduction to Formal Languages and Automata*, Narosa, 2000.
3. G. Lallment, *Semigroups Theory and Applications*, Springer, 1986.

# APMM 323: Linear Programming Techniques (Elective - I)

**Text:**

M. S. Bazaraa, J. J. Jarvis, H. D. Sherali, *Linear Programming and Network Flows*, 4th edition, John Wiley and Sons Publications, New Jersey, 2010.

**COURSE OUTCOMES (CO):**

Operations research helps in solving problems in different environments that needs decisions. This course aims to introduce students to use quantitative methods and techniques for effective decision– making; model formulation and applications that are used in solving business decision problems.

Linear programming problems, Modeling, geometric solutions, examples, solution of systems of linearequations, basic definitions and terminology from graph theory and matrices are the prerequisite of this course.

Upon completion of this course, the student will be able to

| No | Outcome | CO – PSO Mapping |
|---|---|---|
| **CO 1** | Formulate Linear programming problem from a real-life situation. | PSO 1, PSO 5 |
| **CO 2** | Make use of Dual Simplex method to solve the standard linear programming problem. | PSO 5, PSO 8, PSO 9 |
| **CO 3** | To analyze the Karmarker's projective algorithm. | PSO 10 |
| **CO 4** | Solve network flow problems using simplex method. | PSO1, PSO 10 |
| **CO5** | Solve the transportation problem and assignment problems using simplex method. | PSO5, PSO 10 |

## UNIT I

**Linear Programming – Simplex method:**

Convex sets, Convex functions, Extreme points, Simplex method, Basic feasible solutions, geometric motivation of Simplex method, Algebra of Simplex method, Optimality and unboundedness, Tableau format, pivoting, Two phase simplex method, Big M method, Degeneracy.

(Sections 2.4, 2.5, 2.6, 3.1-3.9, 4.2, 4.3 and 4.6)

## UNIT II

**Duality theory and Sensitivity Analysis:**

Revised Simplex method, Simplex method for bounded variables, Formulation of dual, Primal dualrelationship, the dual simplex method, sensitivity analysis.
(Sections 5.1, 5.2, 6.1, 6.2, 6.4 and 6.7.
*Section 6.3 – 'Economic interpretation of dual' may be given as self-study or assignment.*

## UNIT III

**Complexity of Simplex Algorithm:**

Computational complexity of Simplex algorithm, Karmarker's projective algorithm and its analysis,interior point methods. (Sections 8.1-8.6).

## UNIT IV

**Minimal cost network flows:**

Minimal cost network flow problem, basic definitions and terminology from graph theory and matrices, representation of nonbasic vector in terms of basic vector, Simplex method for network flowproblems, network flow with lower and upper bounds, degeneracy, cycling and stalling.
(Sections 9.1-9.12).

Sections 9.2 and 9.3 are meant for review only.

## UNIT V

**Transportation and Assignment problems:**
Definition of transportation problem, Simplex method for transportation problem, degeneracy, Hungarian Algorithm for assignment problem, transshipment problems.
(Sections 10.1-10.7 and 10.10).

## References:


1. Hamdy A. Taha, *Operations Research*, Fifth edition, Pearson Prentice hall, 2007.
2. S. R. Yadav and A. K. Malik, *Operations Research*, Oxford University Press, 2014.
3. Kanti Swarup, P. K. Gupta, Man Mohan, *Operations Research*, Sultan Chand & Sons, 1978.
4. Sharma J. K., *Operations Research: Theory and Applications*, 5th edition, Macmillan India Limited, 2013.
5. Ravindran, Philips, Solberg, *Operations Research, Principles and Practice*, Second Edition, John Wiley & Sons, 1987.
6. K. V. Mital, C. Mohan, *Optimization Methods in Operations Research and Systems Analysis*, Third Edition, New Age International Publishers, New Delhi, 1996.

# APMM 323: Difference Equations (Elective - I)

**Text:**

Saber N. Elaydi, *An Introduction to Difference Equations*, Third Edition, Springer International Edition, First Indian Reprint, New Delhi, 2008.

**COURSE OUTCOMES (CO):**

After the successful completion of this course on difference equations, the students will be able to

| No | Outcome | CO – PSO Mapping |
|---|---|---|
| **CO 1** | Describe the meaning and need of difference equations in real life. | PSO 4, PSO 5 |
| **CO 2** | Solve linear homogeneous difference equations with constant coefficients. | PSO 5 |
| **CO 3** | Solve a system of linear difference equations. | PSO 7 |
| **CO 4** | Apply the method of Z transforms to solve a difference equation. | PSO 4, PSO 5 |
| **CO 5** | Apply the solution techniques to oscillation theory. | PSO 10 |
| **CO 6** | Analyze the asymptotic behavior of solution of a difference equation. | PSO 8 |

## UNIT I

**Linear Difference Equations of Higher Order:** Difference Calculus, General theory of linear difference equations, linear homogenous equations with constant coefficients, Linear non-homogenous equations, Method of undetermined coefficients. [Chapter 2: Sections 2.1 - 2.4].

## UNIT II

**System of Linear Difference Equation:** Autonomous (time invariant) systems, the basic theory, The Jordan form: Autonomous (time-invariant) systems, Linear Periodic Systems. [Chapter 3: Sections 3.1 - 3.4].

## UNIT III

**The Z-Transform Method:** Definitions and examples, Properties of Z-Transform, The inverse Z-Transform and solutions of difference equations, Power series method, Partial fraction method, inversion integral method. [Chapter 6: Sections 6.1, 6.2].

## UNIT IV

**Oscillation Theory:** Three-term difference equations, Self - adjoint second order equations, nonlineardifference equations. [Chapter 7: Sections 7.1 - 7.3].

## UNIT V

**Asymptotic Behavior of Difference Equations:** Tools of approximations, Poincare's theorem,asymptotically diagonal systems. [Chapter 8: Sections 8.1 - 8.3].

### References:


1. S. Goldberg, *Introduction to Difference Equations*, Dover Publications, 1986.
2. Walter G. Kelley, Allan C. Peterson, *Difference Equations: An Introduction with Applications*, Academic Press, Indian Reprint, New Delhi, 2006.
3. V. Lakshmikantham, Donato Trigiante, *Theory of Difference Equations: Numerical Methods and Applications*, $2^{nd}$ edition, Marcel Dekker, Inc, New York, 2002.
4. Ronald E. Mickens, *Difference Equations*, Van Nostrand Reinhold Company, New York, 1987.
5. Sudhir K. Pundir, Rimple Pundir, *Difference Equations* (UGC Model Curriculum), Pragati Prakashan, $1^{st}$ edition, Meerut, 2006.

# APMM 323: Approximation Theory (Elective - I)

**Text:**

E. W. Cheney, *Introduction to Approximation Theory*, Mc Graw Hill, 1966.

**COURSE OUTCOMES (CO):**

After completing this course, the students will be able to

| No | Outcome | CO – PSO Mapping |
|---|---|---|
| **CO 1** | Explain the existence conditions of approximate and optimal solutions in normed spaces. | PSO 1 |
| **CO 2** | Solve the system of equations using several algebraic algorithms. | PSO 4 |
| **CO 3** | Formulate the Vandermonde matrix for the error computation. | PSO 7 |
| **CO 4** | Explain the idea of best rational approximation. | PSO 7 |
| **CO 5** | Apply Stone approximation theorem, Muntz theorem, Unicity theorem, etc. for getting approximate solutions. | PSO 10 |

## UNIT 1

Metric spaces - An existence Theorem for best approximation from a compact subset; Convexity - Caratheodory's Theorem - Theorem on linear inequalities; Normed linear spaces - An Existence Theorem for best approximation from finite dimensional subspaces - Uniform convexity - Strict convexity. (Sections 1, 2, 5 and 6 of Chapter 1).

## UNIT II

The Tchebycheff solution of inconsistent linear equations - Systems of equations with one unknown - Three algebraic algorithms; Characterization of best approximate solution for m equations in unknowns - The special case; Poly's algorithm. (Section 1, 2, 3, 4 and 5 of Chapter 2).

## UNIT III

Interpolation- The Lagrange formula - Vandermonde's matrix-interpolation; The Weierstrass Theorem

- Bernstein polynomials - Theorem; General linear families - Characterization Theorem- Theorem. (Sections 1, 2, 3 and 4 of Chapter 3).

## UNIT IV

Rational approximation - Conversion or rational functions to continued fractions; Existence of best rational approximation - Extension of the classical Theorem; Generalized rational approximation - the characterization of best approximation - An alternation Theorem - the special case of ordinary rational functions; Unicity of generalized rational approximation. (Sections 1, 2, 3 and 4 of Chapter 5).

## UNIT V

The Stone Approximation Theorem, The Muntz Theorem - Gram's lemma, Approximation in the mean-Jackson's Unicity Theorem - Characterization Theorem, Marksoff's Theorem. (Section 1, 2 and 6 of Chapter 6).

## Reference:

P. J. Davis, *Interpolation and Approximation*, Blaisdell Publication, 1963.

# APMM 324: DIFFERENTIAL GEOMETRY

# (Elective - II)

**Text:**

John A. Thorpe, *Elementary Topics in Differential Geometry*, Springer Verlag.

**COURSE OUTCOMES (CO):**

After the completion of this course, the student will be able to

| No | Outcome | CO – PSO Mapping |
|---|---|---|
| **CO1** | Describe the level curves of surfaces in any finite dimension. | PSO 2, PSO 3 |
| **CO2** | Analyze the geometrical smoothness of curves in any finite dimension. | PSO 6 |
| **CO3** | Compute geometrical parameters of curves and surfaces like arc length, curvature, etc. | PSO 5, PSO 6 |
| **CO4** | Demonstrate an advanced level understanding of functions of several variables. | PSO 6 |
| **CO5** | Identify the interplay between Advanced Mathematics and Theoretical Physics through Linear Algebra and Calculus. | PSO 1, PSO 4, PSO 8 |

## UNIT 1

Graphs and level sets, Vector fields, Tangent Spaces. (Chapter 1, 2, 3 of Text).

## UNIT II

Surfaces, Vector fields on surfaces, Orientation, The Gauss map (Chapter 4, 5, 6 of Text).

## UNIT III

Geodesics, Parallel transport (Chapter 7, 8 Text).

## UNIT IV

The Weingarten map, Curvature of plane curve. (Chapter 9, 10 of Text).

## UNIT V

Arc length, Line integral, Curvature of surfaces. (Chapter 11, 12 of Text, except the proofs of Theorem 1, Theorem 2 of Chapter 11 and Theorem 1 of Chapter 12).

**References:**

1. I. Singer and J. A. Thorpe, *Lecture notes on Elementary Topology and Geometry*, Springer- Verlag.

2. M. Spivak, *Comprehensive introduction to Differential Geometry* (Vol. 1 to 5), Publish or Perish Boston.

# APMM 324: Advanced Topics in Graph Theory (Elective - II)

**Text:**

Gary Chartrand and Ping Zhang, *Introduction to Graph Theory,* Tata Mc Graw Hill, 2006.

**COURSE OUTCOMES (CO):**

Upon completion of the course, the students will be able to

| No | Outcome | CO – PSO Mapping |
|---|---|---|
| **CO 1** | Describe the basic concepts of graphs, directed graphs and weighted graphs. | PSO 1, PSO 4 |
| **CO 2** | Model Isomorphism of graphs and study Eulerian and Hamiltonian graphs. | PSO 5 |
| **CO 3** | Interpret the problems of Matchings and Factorization. | PSO 4, PSO 5 |
| **CO 4** | Summarize the vertex and edge coloring of graphs. | PSO 7 |
| **CO 5** | Apply the knowledge of graphs to solve the real-life problems. | PSO 10 |

An overview of the concepts Graphs, Simple graphs, multi graphs, connected graphs, Degree of avertex, Trees, Eulerian graphs, Hamilton graphs, Centre of a graph.

## UNIT I

**Solvability:** Definition of isomorphism, Isomorphism as a relation, Graphs and groups, Reconstruction and solvability. (Sections 3.1, 3.2, 3.3, 3.4).

## UNIT II

**Connectivity:** Cut-vertices, Blocks, Connectivity, Eulerian graphs, Hamilton graphs, Hamilton walksand numbers. (Sections 5.1, 5.2, 5.3, 6.1, 6.2 and 6.3).

## UNIT III

**Matchings and Factorization:** Strong diagraphs, Tournaments, matching, Factorization (Sections 7.1, 7.2, 8.1 and 8.2).

## UNIT IV

**Graph Coloring:** The Four-color problem, Vertex coloring, Four Color theorem, Edge Coloring, Ramsey number of graphs, Turan's Theorem. (Sections 10.1, 10.2, 10.3, 11.1 and 11.2).

## UNIT V

**Distance in Graphs:** The centre of a graph, Distant vertices, locating numbers, Detour and directed distance. (Sections 12.1, 12.2, 12.3, 12.4, 12.5 and 12.6).

**References:**


1. Vasudev C., *Graph Theory Applications*, New Age India Publication, 2006.
2. West D. B., *Introduction to Graph Theory*, Pearson Education Inc, 2001.
3. Bondy and Murthy, *Graph Theory with Applications*, The Macmillan Press Limited, 1976.
4. Chartrand G. and L. Lesniak, *Graphs and Diagraphs*, Prindle, Weber and Schmidt, Boston, 1986.
5. Garey M. R., D. S. Johnson, *Computers and Intractability*, A Guide to the Theory of NP-Completeness, Freeman, San Francisco, 1979.
6. Harary F., *Graph Theory*, Addison - Wesley, 1969.
7. K. R. Parthasarathy, *Basic Graph Theory*, Tata Mc Graw-Hill, New Delhi, 1994.
8. Suresh Singh G., *Graph Theory*, PHI Learning Private Limited, 2010.

# APMM 324: Mechanics (Elective - II)

**Text:**

Herbert Gioldstein, *Classical Mechanics,* Addison Wesley, 1960.

**COURSE OUTCOMES (CO):**

Upon completion of this course, the student will be able to

| No | Outcome | CO – PSO Mapping |
|---|---|---|
| **CO 1** | Interpret the dynamics involving a single particle like projectile motion, Simple harmonic motion, pendulum motion and related problems. | PSO 1, PSO 2 |
| **CO 2** | Describe the path described by the particle moving under the influence of central force. | PSO 5 |
| **CO 3** | Apply the concept of system of particle in finding moment inertia, directions of principle axes and consequently Euler's dynamical equations for studying rigid body motions. | PSO 4 |
| **CO 4** | Determine the equation of motion for mechanical systems using the Lagrangian and Hamiltonian formulations of classical mechanics. | PSO 5, PSO 7 |
| **CO 5** | Evaluate canonical equations using different combinations of generating functions and subsequently developing Hamilton Jacobi method to solve equations of motion. | PSO 7, PSO 10 |

**UNIT I**

**Lagrangian Formulations:** Mechanics of a particle, Mechanics of a system of particles, Constraints, D'Alembert's principles and Lagange's Equations, Velocity dependent potentials and dissipation functions, Simple applications of Lagrangian formulation. (Chapter 1 of Text).

**UNIT II**

**Lagrangian Equation:** Hamilton's principle, Derivation of Lagrange's equation, Some techniques of Calculus of Variation, Extension of Hamilton principle, Conservation Theorems. (Sections: 2.1, 2.2, 2.3, 2.4 and 2.6 of Text).

**UNIT III**

**Central Force problem and reduction:** The two body Central force problem, Reduction to equivalent one body problem equation of notation, the equivalent one-dimensional problem, The Virial Theorems, the differential equations for the orbits, The Kepler problem. (Sections3.1 to 3.6 of Text).

**UNIT IV**

**Cayley Klein parameters:** The Kinematics of a rigid body motion, the independent coordinates of a rigid body Orthogonal transformations, The Eulerian angles, The Cayley Klein parameters, Euler's Theorem on the motion of a rigid body, The Coriolis force. (Sections 4.1, 4.2, 4.4, 4.5, 4.6, 4.9 of Text).

### UNIT V

Tensor Dynamics: The rigid body equations of motion, Angular momentum, Tensor and dynamics, The inertia tensor. (Sections 5.1 to 5.6 of Text).

### Reference:

1. Synge J. L. and Griffith B. A., *Principles of Mechanics*, 3rd edition, Mc Graw-Hill, 1959.

# APMM 324: Theory of Wavelets (Elective - II)

## Text:

Michael Frazier, *An Introduction to Wavelets through Linear Algebra*, Springer, 1999.

## COURSE OUTCOMES (CO):

Prerequisites: Linear Algebra, Discrete Fourier Transforms, elementary Hilbert Space theorems.

Upon completion of this course, the student will be able to

| No | Outcome | CO – PSO Mapping |
|---|---|---|
| **CO 1** | Describe the properties of various scaling functions and their wavelets. | PSO 2, PSO 8 |
| **CO 2** | Explain the properties of multiresolution analysis. | PSO 7, PSO 8 |
| **CO 3** | Demonstrate the applications of orthonormal sets. | PSO 3, PSO 4 |
| **CO 4** | Construct the wavelets using iterative procedure. | PSO 3, PSO 4 |
| **CO 5** | Apply the concept of wavelets in solving various problems like image compression, denoising etc. | PSO 6 |

## UNIT I

**Introduction to Wavelets:** Construction of Wavelets on $Z_n$ the first stage. (Section 3.1).

## UNIT II

**Construction of wavelets:** Construction of Wavelets on $Z_n$ the iteration sets, Examples - Shamon,Daubechies and Haar. (Sections: 3.2 and 3.3).

## UNIT III

**Orthonormal sets:** I_2 (Z), Complete Orthonormal sets, $L^2[-\pi, \pi]$ and Fourier. (Sections 4.1, 4.2 and 4.3).

## UNIT IV

**Fourier Transform:** Fourier Transforms and convolution on I_2 (Z), First stage wavelets on Z. (Section: 4.4 and 4.5).

## UNIT V

**The iteration step for wavelets on *Z*:** Examples, Shamon Haar and Daubechies.

## References:


1. Mayor, *Wavelets and Operators*, Cambridge University Press, 1993.
2. Chui C, *An Introduction to Wavelets*, Academic Press, Boston, 1992.

# SEMESTER IV

# APMM 421: Theory of Linear Operators

**Text:**

Erwin Kreyszig, *Introductory Functional Analysis with Applications*, John Wiley & Sons, 2007.

**COURSE OUTCOMES (CO):**

After completing this course on Theory of Linear operators, the student will be able to,

| No | Outcome | CO – PSO Mapping |
|---|---|---|
| **CO 1** | Analyze the spectrums of bounded linear operators, compact linear operators on normed spaces. | PSO 1, PSO 2, PSO 9 |
| **CO 2** | Analyze the spectrum of bounded self-adjoint linear operators on Hilbert spaces. | PSO 1, PSO 2 |
| **CO 3** | Solve operator equations involving compact linear operators. | PSO 3, PSO 4 |
| **CO 4** | Apply Banach fixed point theorem to linear equations, differential equations and integral equations. | PSO 7, PSO 10 |
| **CO 5** | Summarize the concept of unbounded linear operators, and spectrum. | PSO 10 |
| **CO 6** | Demonstrate the spectrum of unitary and self-adjoint operators. | PSO 8 |

## UNIT I

**Spectral theory of linear operators in normed spaces:**

Spectral theory in finite dimensional normed spaces, basic concepts, point spectrum, continuous spectrum, residual spectrum, spectral radius, spectral properties of bounded linear operators, propertiesof resolvent and spectrum, spectral mapping theorem for polynomials, Banach Algebras, properties ofBanach algebra. [Sections: 7.1 - 7.7].

## UNIT II

**Compact linear operators on normed spaces and their spectrum:**

Definition of compact linear operator, compactness criterion, sequence of compact linear operators, total boundedness, separability of range, compact extension, spectral properties, compactness of product, operator equations involving compact linear operators. [Sections 8.1 - 8.5].

## UNIT III

**Spectral theory of bounded self-adjoint linear operators on Hilbert spaces:**

Hilbert-adjoint operator, resolvent set, spectrum, further special properties, residual spectrum, positive operators, product of positive operators, square roots of positive operators, projection operators, positivity, norm, product of projections, sum of projections, difference of projections. [Sections: 9.1 -9.6].

## UNIT IV

**Unbounded linear operators in Hilbert spaces:**

Hellinger - Toeplitz theorem on boundedness, Hilbert-adjoint operator, symmetric and self-adjoint linear operators, Inverse of the Hilbert-adjoint operator, closed linear operators and closures, Hilbert adjoint of the closure, regular values, spectral representation of unitary operators, spectral representation of self-adjoint linear operators, multiplication and differentiation operator. [Sections 10.1 - 10.7].

## UNIT V

**Fixed point theory and applications:**

Banach fixed point theorem, Applications of Banach Fixed Point theorem to linear equations, to differential equations and to integral equations. [Sections 5.1 - 5.4].

**Basics of approximation theory in normed and Hilbert spaces:**

Approximation in normed spaces, Uniqueness, Strict Convexity, Uniform Approximation, Chebyshev Polynomials, approximation in Hilbert spaces. [Sections 6.1 - 6.5].

## References:

1. Bryan Rynne, M. A. Youngson, *Linear Functional Analysis*, Springer, 2008.
2. Rajendra Bhatia, *Notes on Functional Analysis*, Hindustan Book Agency, 2015.
3. M. Thamban Nair, *Functional Analysis: A First Course*, Prentice Hall of India Pvt. Ltd., 2001.
4. Walter Rudin, *Functional Analysis*, II Edition, Publisher: Tata Mc Graw Hill, 1991.
5. B. V. Limaye, *Linear Functional Analysis for Scientists and Engineers*, Springer, Singapore, 2016

# APMM 422: Advanced Complex Analysis

**Text:**

John B. Conway, *Functions of one Complex Variables*, (2nd Edition) Springer Verlag, New York, 1973.

**COURSE OUTCOMES (CO):**

Upon completion of this course, the student will be able to:

| No | Outcome | CO – PSO Mapping |
|---|---|---|
| CO 1 | Recognize the compactness and convergence in the space of analytic functions. | PSO 4, PSO 7 |
| CO 2 | Explain Weierstrass factorization Theorem. | PSO 8 |
| CO 3 | Describe the Mittag Leffler theorem. | PSO 8 |
| CO 4 | Explain the principle of Analytic Continuation and explain Schwarz Reflection Principle and Monodromy theorem. | PSO 7, PSO 10 |
| CO 5 | Demonstrate the Harmonic functions on a disc and its related results. | PSO 1 |

## Unit I

Compactness and convergence in the space of analytic functions. The space C(G, Ω), Space of analytic functions, Riemann Mapping Theorem (Lemma 4.3, statement only). (Chapter 7: Sections 1, 2 and 4).

## Unit II

Weierstrass factorization Theorem, Factorization of sine function. The Gamma function (Only statements of Theorem 7.15 and Lemmas 7.16, 7.17, and 7.19) (Chapter 7: Section 5, 6 and 7).

## Unit III

Riemann Zeta function, Runge's Theorem, Simple Connectedness, Mittag Leffler's Theorem.(Chapter 7: Section 8 (excluding 8.13 and 8.14) and Chapter 8).

## Unit IV

Schwarz Reflection Principle, Analytic continuation along a path, Monodromy theorem. (Chapter 9: Sections 1, 2 and 3).

## Unit V

Basic properties of harmonic functions, Harmonic functions on a disc, Jensen's formula, The genusand order of an entire function, Hadamard factorization Theorem.

(Chapter 10: Sections 1(Maximum Principle- second version - statement only), 2; Chapter 11: Sections 1, 2 (Theorem 2.6 statement only), 3 (Lemma 3.1 statement only).

## References

1. L.V. Ahlfors, *Complex Analysis,* Mc-Graw Hill, 1966.
2. S. Lang, *Complex Analysis,* Mc-Graw Hill, 1998.
3. S. Ponnusamy & H. Silverman, Complex Variables with Applications, Birkhauser, 2006.
4. H. A. Priestley, Introduction to Complex Analysis, Oxford University Press, 1999.
5. V. Karunakaran, Complex Analysis, Narosa Publishing House, 2005.

# APMM 423: Coding Theory (Elective - III)

**Text:**

D. J. Hoffman, Coding Theory the Essentials, Marcel Dekker Inc., 1991.

**COURSE OUTCOMES (CO):**

Upon completion of this course, the student will be able to

| No | Outcome | CO – PSO Mapping |
|---|---|---|
| **CO 1** | Demonstrate basic techniques of algebraic coding theory like matrix encoding, polynomial encoding and decoding by coset leaders etc. | PSO 2, PSO 5 |
| **CO 2** | Distinguish different types of codes like linear, perfect, Hamming, Golay, cyclic and BCH codes. | PSO 5 |
| **CO 3** | Apply algebraic coding theory to solve the real-world problems. | PSO 7 |
| **CO 4** | Explain more about cyclic linear codes and dual cyclic codes. | PSO 7, PSO 10 |
| **CO 5** | Explain the ideas of decoding of 2 errors correcting BCH codes. | PSO 10 |

**UNIT I**

**Detecting and correcting error patterns:** Information rate, the effects of error detection and correction, Finding the most likely code word transmitted, Weight and distance, MLD, Error detecting and correcting codes. (Chapter 1 of the Text).

**UNIT II**

**Linear codes:** bases for $C = \langle S \rangle$ and $C^{\perp}$, generating and parity check matrices, Equivalent codes, Distance of a linear code, MLD for a linear code, Reliability of IMLD for linear codes. (Chapter 2 of the Text).

**UNIT III**

**Perfect codes:** Hamming code, Extended codes, Golay code and extended Golay code, Reed - Muller Codes. (Chapter 3 sections: 1 to 8 of the Text).

**UNIT IV**

**Cyclic linear codes:** Polynomial encoding and decoding, Dual cyclic codes. (Chapter 4 and Appendix A of the Text).

**UNIT V**

**Cyclic Hamming Codes:** BCH Codes, Cyclic Hamming Code, Decoding 2 error - correcting BCH codes. (Chapter 5 of text).

## References:

1. E. R. Berlekamp, *Algebriac Coding Theory*, Mc-Graw Hill, 1968.
2. P. J. Cameron and J. H. Van Lint, *Graphs*, *Codes and Designs*, London Mathematical Society, 2016.
3. H. Hill, *A First Course in Coding Theory*, Oxford University Press, 1986.

# APMM 423: Spectral Graph Theory (Elective - III)

**Text:**
Bogdan Nica, *A Brief Introduction to Spectral Graph Theory*, European Mathematical SocietyPublishing House, 2018.

**COURSE OUTCOMES (CO):**
Upon the completion of this course, the student will be able to

| No | Outcome | CO – PSO Mapping |
|---|---|---|
| **CO 1** | Explain the necessity and advantage of different matrix representations and algebraic parameters of a graph. | PSO 6 |
| **CO 2** | Find the chromatic number, independence number and eigen values of a graph. | PSO 2, PSO 6 |
| **CO 3** | Apply the famous Turan's theorem. | PSO7, PSO 10 |
| **CO 4** | Find the spectral bounds for a graph. | PSO 5 |
| **CO 5** | Apply the concepts of spectral graph theory to solve the real-life problems. | PSO 6, PSO 7 |

## UNIT I

**Eigenvalues of a Graph:** A quick review of Chapter 1, Invariants, Chromatic number and independence number (Section 2.1 of Chapter 2), Eigenvalues of graphs, Adjacency and Laplacian eigenvalues, First properties, First examples. (Chapter 7).

## UNIT II

**Computation of Eigenvalues:** Eigenvalue computations, Cayley graphs and bi-Cayley graphs of abelian groups, Strongly regular graphs, Two gems, Design graphs. (Chapter 8) (Except Example 8.10 and Example 8.23).

## UNIT III

**Spectrum of different types of graphs:** Largest eigenvalues, Extremal eigenvalues of symmetric matrices, Largest adjacency eigenvalue, The average degree, A spectral Turán theorem, Largest Laplacian eigenvalue of bipartite graphs, Sub graphs, Largest eigenvalues of trees. (Chapter 9).

## UNIT IV

**Different types of Eigenvalues:** More eigenvalues, Eigenvalues of symmetric matrices: Courant–Fischer, A bound for the Laplacian eigenvalues, Eigenvalues of symmetric matrices:
Cauchy and Weyl, Sub graphs (Chapter 10).

## UNIT V

**Spectral bounds:** Chromatic number and independence number, Isoperimetric constant, Edge counting. (Chapter 11).

**References:**

1. D. Cvetkovic, M. Doob, H. Sachs, *Spectra of Graphs - Theory and Applications*, Academic Press, New York, 1980.

2. Andries E. Brouwer, William H. Haemers, *Spectra of graphs* – Monograph, Springer 2011.

# APMM 423: Statistical Methods and Nonlinear Optimization (Elective – III)

**Texts:**

1. Sheldon M. Ross, *Stochastic Processes*, 2nd Edition, John Wiley and sons, 1996.
2. J. K. Sharma, *Operations Research Theory and Applications*, 2nd Edition, Macmillan, 1996.
3. M. S. Bazaraa, H. D. Sherali, C. M. Shetty, *Nonlinear Programming, Theory and Algorithms*, 3rd Edition, John Wiley and Sons, 2006.

**COURSE OUTCOMES (CO):**

Upon completion of the course, the students should be able to

| No | Outcome | CO – PSO Mapping |
|---|---|---|
| **CO 1** | Apply Markov Chains to solve practical problems. | PSO 4, PSO 5, PSO 9 |
| **CO 2** | Classify different queuing models. | PSO 5, PSO 9 |
| **CO 3** | Explain the basics of nonlinear optimization Problems. | PSO 5, PSO 9 |
| **CO 4** | Analyze and interpret nonlinear optimization problems. | PSO 10 |
| **CO 5** | Demonstrate the concept of penalty functions and Barrier methods and their geometric interpretation. | PSO 10 |

## UNIT I

**Markov Chains:**

Introduction and examples, Champman - Kolmogrov equations and Classification of states, Limit theorems, transition among classes, transient states, Branching process, Applications of Markov chains. (Sections 4.1, 4.2, 4.3, 4.4, 4.5, 4.6 of Text 1).

## UNIT II

**Queuing Theory:**

Introduction, Essential features of a queuing system, Performance measurers of a queuing system, probability distributions in queuing system, Classification of queuing models and their solutions -Single server queuing models. (Sections 16.1 - 16.6 of Text 2).

## UNIT III

**The Fritz John and Karush – Kuhn - Tucker Optimality conditions:** Unconstrained problems, Problems having inequality constraints, Problems having inequality and equality constraints, Second order necessary and sufficient optimality conditions for constrained problems, Lagrangian dual problem and geometrical interpretation. (Sections 4.1, 4.2, 4.3, 4.4 and 6.1 of Text 3).

## UNIT IV

Unconstrained Optimization: Line search without using derivatives: Uniform search, golden section search and Fibonacci method, Line search using derivatives: Bisection search method, Newton's Method, Multidimensional search without using derivatives: Hooke and Jeeves. Rosenbrock's methods and their convergence, Methods using conjugate directions: Conjugate gradient methods, Fletcher Reeves, conjugate directions, Steepest descent algorithms. (Sections 8.1, 8.2, 8.5, 8.8 of Text 3).

Section 8.6 may be given as self study or assignment.

## UNIT V

Penalty Barrier Methods and Methods of feasible directions: Concept of penalty functions, geometric interpretation, exterior penalty function methods, Barrier function methods, Methods of feasible directions for nonlinear programming, quadratic programming, gradient projection method of Rosen. (Sections 9.1, 9.2, 9.4, 10.4 and 10.5 of Text 3).

## References:


1. Rohatgi V. K., *An Introduction to Probability Theory and Mathematical Statistics*, Wiley and Sons, 1985.
2. Rao, C. L., *Linear Statistical Inferences and Applications*, Wiley and Sons, 1974.
3. Mood Ali, Gray Bill Fhardoes D. C., *Introduction to the Theory of Statistics*, Mc Graw Hill International, 3rd Edition, 1972.
4. H. A. Taha, *Operation Research - An introduction*, Prentice Hall, 7th Edition, 2006.
5. D. M. Simmons, *Nonlinear Programming for Operations Research*, Prentice Hall, 1975.

# APMM 423: Category Theory (Elective - III)

**Text:**
S. Maclane, *Categories for the working Mathematician*, Springer, 1971.

**COURSE OUTCOMES (CO):**
Upon completing this course, the student will be able to

| No | Outcome | CO – PSO Mapping |
|---|---|---|
| **CO 1** | Understand the basics of categories and functors. | PSO 2, PSO 4 |
| **CO 2** | Distinguish different types of categories and functors. | PSO 6, PSO 7 |
| **CO 3** | Construct new categories from the existing ones like product category, quotient category etc. | PSO 6, PSO 7 |
| **CO 4** | Explain the concept of transformations of adjoints and compositions of adjoints. | PSO 7, PSO 8 |
| **CO 5** | Analyze equalizers, adjoints and subcategories and their properties. | PSO 10 |

## UNIT-I

Categories, Functors and Natural Transformations: Axioms for categories, categories, Functors. Natural Transformations, Mobics, Epis and Zeros Foundations, Large Categories, Hom-sets.

## UNIT II

Constructions on categories: Duality Contravariance and opposites, Products of Categories. Functor Categories, The category of all categories, Comma categories, Graphs and Free categories, Quotient Categories.

## UNIT III

Universals and Limits: Universal Arrows, Yoneda Lemma Coproduces and Colimits, Products and Limits, Categories with Finite products, Groups in categories.

## UNIT IV

Adjoints: Adjunctions, Examples of adjoints, Reflective subcategories, Equivalence of categories, Adjoints for pre orders, Cartesian closed categories, Transformations of adjoints, Compositions of adjoints.

## UNIT –V

Limits: Creation of limits by products and Equalizers, Limits with parameters, Preservation of Limits, Adjoints on Limits, Freyd’s Adjoint Functor Theorem, Sub objects and Generation, The Special Adjoint Functor Theorem, Adjoint in Topology.

**References:**

1. M. A. Arbib and E. G. Maneswarrows, *Structures and Functors, The categorical Imperative*, Academic Press, 1975.
2. H. Herrlich and G. E. Strecker, *Category Theory*, Allyn & Bacon, 1973.
3. M. Barmand, C. Wells, *Category Theory for Computer Science*, Prentice Hall, 1990.
4. F. Borceux, *Handbook of Categorical Algebra*, Vol. I, II, III, Cambridge University Press, 1994.
5. P. Frevd, *Abelian Categories*, Harper & Row, 1964.
6. R. F. C. Walters, *Categories and Computer Science*, Cambridge University Press, 1991.

# APMM 424: Artificial Neural Networks and Machine Learning (Elective - IV)

## Texts

1. Jacek M. Zurada, *Introduction to Artificial Neural Systems*, PWS Publishing Company, 1995.
2. Simon Haykin, *Neural Networks: A Comprehensive Foundation*, Macmillan College Publishing Company, 1994.
3. Bishop C., *Pattern Recognition and Machine Learning*, Berlin: Springer-Verlag, 2006.

## COURSE OUTCOME (CO):

Upon completion of this course, the student will be able to

| No | Outcome | CO – PSO Mapping |
|---|---|---|
| **CO 1** | Analyze the role of neural networks in machine learning process. | PSO 2, PSO 3 |
| **CO 2** | Apply the artificial neural networks in real life situations. | PSO 7, PSO 8 |
| **CO 3** | Make use of different artificial neural network models. | PSO 8 |

## UNIT I

A Neural Network, Human Brain, Models of a Neuron, Biological Neuron, Artificial Neural Model, McCulloch Pitts Model, Perceptron, Types of activation functions, Neural Network Architectures, Feed forward and Recurrent Networks, Linear Separability, Non-Linear Separable Problem. XOR Problem, Multilayer Networks.

## UNIT II

Supervised Learning, Unsupervised Learning, Learning Algorithms, Amari's general Learning Rule, Hebbian, Perceptron Learning Algorithm, Perceptron Convergence Theorem, Multi- layered Network Architecture, Back propagation Learning Algorithm, Network Pruning Techniques, Virtues, and Limitation of Back Propagation Learning, Accelerated Convergence.

## UNIT III

Learning from Examples, Statistical Learning Theory, Support Vector Machines, SVM application to Image Classification, Radial Basis Function Regularization theory, Generalized RBF Networks, Learning in RBFNs, RBF application to face recognition.

## UNIT IV

Associative Memory: Associative Learning Attractor Associative Memory, Linear Associative memory, Hopfield Network, application of Hopfield Network, Brain State in a Box neural Network, Simulated Annealing, Boltzmann Machine, Bidirectional Associative Memory.

## UNIT V

Self-organization Feature Map: Maximal Eigenvector Filtering, Extracting Principal Components, Generalized Learning Laws, Vector Quantization, Self - organization Feature Maps, Application of SOM.

Machine learning basics: capacity, overfitting and underfitting, hyperparameters and validation sets, bias & variance; PAC model; Rademacher complexity; growth function; VC – dimensions.

## References:

1. Mohamad H. Hassoun, *Fundamentals of Artificial Neural Networks*, The MIT Press, 1995.
2. Satish Kumar, *Neural Networks*, Tata Mc Graw Hill, 2013.
3. Laurene Fausett, *Fundamentals of Neural Networks: Architectures, Algorithms and Applications*, Prentice Hall International, 1994.
4. Ethem Alpaydin, *Introduction to Machine Learning*, 2nd Edition, MIT Press, 2004.

# APMM 424: Calculus of Variations and Linear Integral Equations (Elective - IV)

**Texts:**

1. M. L. Krasnov, G. I. Makarenko, A. I. Kiselev, *Problems and Exercises in the Calculus of Variations*, MIR Publishers, 1975.
2. Ram P. Kanwal, *Linear Integral Equations: Theory and Technique*, Academic Press, New York, 1971.

**COURSE OUTCOMES (CO):**

This course is intended to prepare the student with mathematical tools and techniques that are required in advanced courses offered in the applied mathematics and engineering programs. The objective of this course is to enable students to study the Calculus of Variation technique and for solving integral equations and apply these ideas to solve ordinary differential equations.

After the completion of this course, the students will be able to

| No | Outcome | CO – PSO Mapping |
|---|---|---|
| **CO1** | Solve the Euler's Equation. | PSO 2, PSO 4 |
| **CO2** | Solve the problems using Ritz's and Kantorovich's methods. | PSO 4 |
| **CO3** | Apply the approximation method to solve the integral equations. | PSO 2, PSO 6 |
| **CO4** | Solve the linear integral equations of first and second type (Volterra and Fredhlom). | PSO 6, PSO 7 |
| **CO5** | Solve the ordinary differential equations using Green's functions. | PSO 6, PSO 10 |

**UNIT I**

The Functional - The variation of a functional and its properties, Euler's equation, Generalizations of the elementary problem of the Calculus of Variations, Invariance of Euler's Equation, Field of Extremals, Sufficient conditions for the Extremum of a Functional, Conditional Extremum, Moving Boundary Problems.

(Chapter 2: Sections 3 - 12 of Text 1).

**UNIT II**

Euler's Finite difference method – problems, Ritz's and Kantorovich's methods, Variational Methods for finding eigen values and eigen Functions.

(Chapter 3: Section 13 -15 of Text 1).

## UNIT III

Introduction: Definition, regularity conditions, special kinds of kernels, eigenvalues and eigenvectors, convolution integrals, the inner product of two functions. Integral equations with separable kernels: Reduction to a system of equations, examples, Fredholm alternative, examples, an approximation method.
(Chapter 2: Sections 2.1 – 2.5 of Text 2).

## UNIT IV

Method of successive approximations: Iterative schemes, examples, Volterra integral equations, examples, some results about the resolvent kernel. Classical Fredholm theory: The method of solution of Fredholm, Fredholm's first theorem, examples, Fredholm's second theorem, Fredholm's third theorem.
(Chapter 3: Section 3.1 – 3.5, Chapter 4: Section 4.1 – 4.5 of Text 2).

## UNIT V

Applications to ordinary differential equations: Initial value problems, boundary value problems, examples, Dirac's delta function, Green's function approach, examples, Green's function for nth order ordinary differential equations, modified Green's function, examples.
(Chapter 5: Section 5.1 – 5.8 of Text 2).

**References**:

1. Edouard Goursat, *Integral Equations, Calculus of Variations: A Course in Mathematical Analysis* Vol. III, Part Two, Dover, 1964.
2. Bernard Dacorogna, *Introduction to the Calculus of Variations*, Imperial College Press, World Scientific, 2004.
3. Bolza, Oskar, *Lecture Notes on Calculus of Variations*, Biblio Bazar, 2009.
4. Mariano Giaquinta, *Topics in Calculus of Variations*, Springer, 1989.
5. Bruce Van Brunt, *The Calculus of Variations*, Springer, 2004.
6. Harry Hochstadt, *Integral Equations*, Wiley, 1973.
7. T A Burton, *Volterra Integral and Differential Equations*, Elsevier, 1983.
8. Anderi D Polyanin, Alexander V Manzhirov, *Handbook of Integral Equations*, CRC Press, 1998.

# APMM 424: Commutative Algebra (Elective - IV)

**Text:**
N. S. Gopalakrishan, *Commutative Algebra*, Oxonian Press, 1986.

**COURSE OUTCOMES (CO):**
After completing this course successfully, the student will be able to

| No | Outcome | CO – PSO Mapping |
|---|---|---|
| **CO 1** | Analyze the tenser product of modules and Flat modules. | PSO 1, PSO 4 |
| **CO 2** | Apply ideals and local rings. | PSO 4, PSO 6 |
| **CO 3** | Identify different types of modules and their characterizations. | PSO 10 |
| **CO 4** | Apply the concept of Noetherian rings and its extensions. | PSO 1, PSO 7 |
| **CO 5** | Explain about Artinian modules and Dedekind domain. | PSO 6, PSO 8 |

## UNIT I

Modules: Modules, Free projective, Tensor product of modules, Flat modules. (Chapter 1 of Text).

## UNIT II

Ideals and Local Rings: Ideals, Local rings, Localization and applications. (Chapter 2 of Text).

## UNIT III

Noetherian Rings: Noetherian rings, modules, Primary decomposition, Artinian modules. (Chapter 3 of Text).

## UNIT IV

Integral Domains: Integral extensions, Integrally closed domain, Finiteness of integral closure. (Chapter 4 of Text).

## UNIT V

Valuation Rings: Dedekind domain. (Chapter 5 of Text, Theorems 4 and 5 omitted).

**References:**

1. M. F. Atiyah and I. G. MacDonald, *Introduction to Communicative Algebra*, Addison - Wesley, 1994.

3. T. W. Hungerford, *Algebra*, Springer - Verlag, 1974.

# APMM 424: Representation Theory of Finite Groups (Elective - IV)

**Text:**

Walter Ledermann, *Introduction to Group Characters*, Cambridge University Press, 1987.

**COURSE OUTCOMES (CO):**

After the successful completion of this course, the student will be able to

| No | Item | CO – PSO Mapping |
|---|---|---|
| **CO 1** | Explain the structure and characterizations of finite groups. | PSO 4 |
| **CO 2** | Inspect on characterizing finite groups, which are not yet characterized. | PSO 6 |
| **CO 3** | Discuss about commutant algebra and orthogonality relations. | PSO 1, PSO 10 |
| **CO 4** | Analyze the character table of finite groups. | PSO 6 |
| **CO 5** | Explain about Frobenius groups. | PSO 6, PSO 7 |

**UNIT I**

G-module: Characters, Reducibility, Permutation representations, complete reducibility, Schur's Lemma. (Sections 1.1 - 1.7 of Text).

**UNIT II**

The commutant algebra: Orthogonality relations, the group algebra (Sections 1.8, 2.1, 2.2 of Text).

**UNIT III**

Character table: Character of finite abelian groups, The lifting process, Linear characters. (Sections 2.3, 2.4, 2.5, 2.6 of Text).

**UNIT IV**

Induced representations: Reciprocay law, A5, Normal subgroups, Transitive groups, Induced characters of Sn. (Sections 3.1, 3.2, 3.3, 3.4, 4.1, 4.2, 4.3 of Text).

**UNIT V**

Group theoretical applications: Brunside's (p, q) Theorem, Frobenius groups. (Chapter 5 of Text).

**Reference:**

1. S. Lang, *Algebra*, Addison - Wesley, 2002.